# Sea Level Fluctuations in Tórshavn, Preliminary Results

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#### Introduction

The fluctuations of sea level and currents at the Faroe Islands make quite an interesting subject. The position of the Islands, far from the continents, close to a supposed amphidromy and in one of the key areas with respect to the overflow of cold bottom water from the Norwegian and Greenland seas into the Atlantic indicate the theoretical importance, while the very strong tidal currents on the Faroese plateau make obvious the practical importance. However to the authors knowledge very little has been published on the subject, the highlight being a paper by Adam Poulsen dating back to 1906. He analyzed the tide in Tórshavn and his results are reproduced in table I.

The present study concerns itself with tide gauge measurements made in Tórshavn for the most part of 1973. As the work is still in progress, only preliminary results will be discussed. These include a tabulation of Harmonic constants and some investigations into the influence of atmospheric pressure on the sea level.

### Data

The sea level registrations analyzed in this paper cover the period: Jan 1.st 1973 0000 GMT to Oct 19th 1973 2345 GMT sampled at 15 minutes interval. The tide gauge was situated in the harbor of Tórshavn and was owned and supervised by the

Nautical Department of the Danish Meteorological Institute (DMI). The gauge digitized the measured values and stored them on papertapes, which were changed each month by the Harbor master of Tórshavn. Edited data were supplied to the author by the DMI.

The air pressure data also were supplied by the DMI, they had been recorded at the Meteorological station of Tórshavn approximately 2 km's distant from the gauge and had a sampling time interval of 3 hours.

Parts of the analysis require that there be no gaps in the time series. In the sea level series there were two consecutive registrations missing out of a total of about 30.000. The values of these were interpolated and filled in. The meteorological series was much less complete, as about 2 % of the registrations were missing, rather evenly distributed in the period. These values also were interpolated and inserted at those parts of the analysis where it was necessary. It is not felt that these manipulations could have any influence on the results presented.

Methods of analysis

We shall assume the sea level h to vary in time according to the equation

$$h(k) = \sum_{i}^{c} c_{i} \cdot \cos(2 \cdot \pi \cdot f_{i} \cdot k \cdot \Delta t + \varphi_{i}) + h_{p}(k) + h_{R}(k)$$
 (1)

 $\triangle$ t being the sampling time interval and h(k) the height of sea level at the time k  $\cdot$   $\triangle$ t. The first term on the right is the harmonic part containing the astronomical frequencies  $f_i$ .  $h_p$  is the sea level height due to air pressure and  $h_R$  is the rest, which we may here consider as noise.

Various methods exist for extracting the parameters  $f_i$ ,  $c_i$   $\varphi_i$  from a tidal series. The method mainly employed in this work is rather unconventional, firstly as it used the Power Density Spectrum (PDS) — thereby making no assumptions as to what frequencies are present — and secondly because the PDS is estimated by the Maximum Entropy Method (MEM) rather than by the traditional use of either the Fast Fourier Transform or the Autocorrelation function.

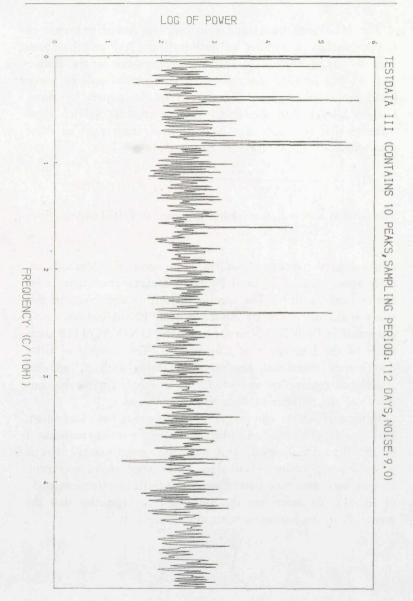


Fig. 1. MEM-spectrum of test series. Vertical scale logarithmic.

The MEM has been used previously in other branches of Geophysics by a number of authors (ref. 2,3) and has been used by Arne Nielsen to identify eigenmodes of the Baltic (ref. 4). No detailed description of the method will be given here; interested readers are referred to the papers by Burg, Lacoss, Ulrych and Andersen (refs. 5 to 10). Suffice it to mention that the PDS may be calculated from a set of filter coefficients  $a_1, a_2, \dots, a_m$  by the formula

$$P(f) = \frac{P_0 \cdot \Delta t}{\left|11 - \sum_{k=1}^{\infty} a_k \cdot \exp(-2 \cdot \pi \cdot i \cdot k \cdot \Delta t \cdot f)\right|^2}, \frac{-1}{2\Delta t} \le f \le \frac{1}{2\Delta t} \quad (2)$$

The filter is a »Prediction« filter designed to fulfill the equation

$$h(k) = a_1 \cdot h(k-1) + \dots + a_m \cdot h(k-m) + r_k$$
 (3)

minimizing the power P<sub>o</sub> of the noise series r<sub>k</sub>. Methods for calculating a<sub>1</sub>,..., a<sub>m</sub> and P<sub>o</sub> from a given time series were constructed by Burg. The specific algorithm employed in this study is the one given by Andersen (ref. 10); it has been programmed in FORTRAN and run on the UNIVAC 1110 computer of the University of Copenhagen. The number of filter coefficients is optional, resolution increasing with m, however experience requires m to be less than one half (preferably one third) of the number of data values analyzed.

Filter coefficients and power of noise having been computed, P(f) may be evaluated and plotted for any wanted frequency f in the Nyquist interval, and harmonic constituents- that is periodic components — will show up as peaks in the spectrum. In this way one may determine the significant frequencies  $f_i$  of eq. (1). To determine the amplitudes  $c_i$  consider that the power  $P_i$  of an harmonic with amplitude  $c_i$  is

$$P_{i} = \frac{1}{2} \cdot c_{i}^{2} = \begin{cases} f_{i} + \Delta f \\ P(f) \cdot df \\ f_{i} - \Delta f \end{cases}$$
 (4)

(Actually there is an additional factor of 2 as both positive and negative frequencies are used. It will be omitted in the formulae

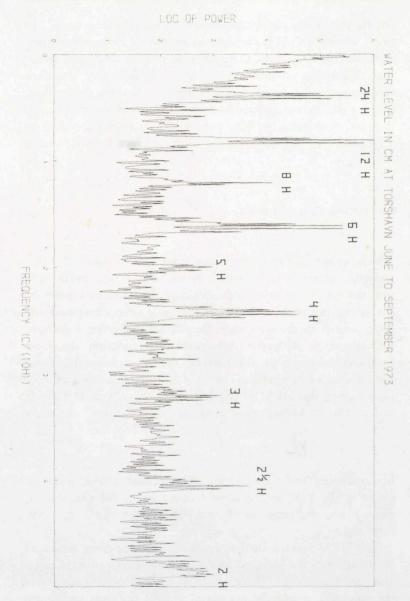


Fig. 2. MEM-spectrum of Sea level fluctuations Tórshavn. Vertical scale logarithmic. Approximate periods of peaks listed.

cited). Hence  $c_i$  can be calculated from the area under the peak. The width  $\triangle f$  is ambiguous, but for an harmonic distinguishable from the noise, the peak is so narrow and pronounced that this makes no problem as long as it is clearly separated from other peaks. In practise it is not very feasible determining the integral by numerical quadrature, rather one may employ the fact, mentioned by Burg (ref. 6) that a significant peak at a frequency  $f_i$  will have a definite shape given by the formula

$$P(f) = \frac{P(f_i)}{1 + (f - f_i)^2 / \delta^2}$$
 (5)

Integration then yields

$$\frac{1}{2} c_i^2 = \pi \cdot P(f_i) \cdot \delta \tag{6}$$

which permits  $c_i$  to be determined from  $f_i$ ,  $P(f_i)$  and  $\delta$ . Computational methods which, from a given set of filter coefficients, find the  $f_i$ 's by binary search, evaluate  $P(f_i)$  and determine  $\delta$  by fitting the peak to equation (5) have been developed and programmed to the computer; details will be given elsewhere

In common with the Autocorrelation function the filter coefficients contain no information on the phase of harmonic constituents of the series. The  $\varphi_i$ 's of eq(1) may however be found using the MEM in the following way. Assume the  $f_i$ 's and  $c_i$ 's to be known, then construct the two new time series

By analyzing these two series as described above, one gets peaks in the PDS at the same frequencies as for the original series h and the ratios of the new amplitudes to the  $c_i$ 's will yield the  $\varphi_i$ 's.

The MEM is very demanding both in computing time and in storage and the demand increases with length of data series and length of filter. Therefore even on a large computer like the one used, it may be necessary to split the data series into separate parts for analysis. If the series is to be split into n

| SYMBOL   | 01  | P <sub>1</sub> | К1  | 2MS <sub>2</sub> | N <sub>2</sub> | M <sub>2</sub> | S2   | K <sub>2</sub> | M <sub>4</sub> |      |
|----------|-----|----------------|-----|------------------|----------------|----------------|------|----------------|----------------|------|
| AMPLIT.  | 7.3 | 1.5            | 4.3 | 0.5              | 1.7            | 10.4           | 5.6  | 0.9            | 2.8            | cm   |
| GR.P.LAG | 53  | 143            | 142 | 97               | -168           | -159           | -142 | -149           | 72             | deg. |

TABLE II Results of analysis of a constructed test time series consisting of 9 harmonics and white noise of power 9  ${\rm cm}^2$ 

| Values use struc |           | Values found by ana-<br>lysis with MEM |           |  |  |
|------------------|-----------|--|-----------|--|--|
| Frequency        | Amplitude | Frequency                              | Amplitude |  |  |
| c/h              | cm        | c/h                                    | cm        |  |  |
| .00100           | 3.00      | .00101                                 | 2.96      |  |  |
| .00300           | 3.00      | .00294                                 | 2.97      |  |  |
| .01000           | 3.00      | .00995                                 | 2.94      |  |  |
| .03840           | 4.00      | .03847                                 | 3.88      |  |  |
| .04160           | 12.00     | .04162                                 | 11.97     |  |  |
| .08050           | 11.00     | .08045                                 | 11.19     |  |  |
| .08330           | 3.00      | .08343                                 | 2.82      |  |  |
| .16080           | 3.00      | .16087                                 | 2.80      |  |  |
| .24080           | 3.00      | .24092                                 | 2.97      |  |  |

parts, this may be done taking the data values modulo n thus obtaining n samples of the time series with sampling intervals  $n \cdot \triangle t$  and phase shifted.

As the MEM to the authors knowledge, has not been used previously in a quantitative study of the Astronomic tides it may be reasonable to demonstrate its capability in this respect. A test series was constructed of the form

$$X(k) = \sum_{i} c_{i} \cdot \cos(2 \cdot \pi \cdot f_{i} \cdot k \cdot \Delta t + \varphi_{i}) + r_{k}$$
 (8)

with known values for  $c_i$ ,  $f_i$  and  $\varphi_i$ . Nine harmonics were used with frequencies usually found in tidal series. The known values for amplitude, frequency and phase are listed in columns 1 and 2 of table II.  $r_k$  was a generated random series — white noise — which in this case had a power of 9. The series had a length corresponding to 112 days of quarter hourly sampling. After construction the timeseries was digitized to integer values to compare as closely as possible to the actual sea level series. The series was then split modulo 4 and one of the parts was analyzed by the methods described. The MEM spectrum of this testdata is shown in fig. 1. The nine harmonics are clearly distinguishable on the figure and the values of frequency and amplitude determined by analysis are listed in columns 3 and 4 of table II to be compared to the previous two columns.

#### Results

Harmonic constants The sea level data were analyzed by the methods listed in the previous part. Fig. 2 shows the MEM spectrum in the total Nyquist range while figs. 3 and 4 show parts of it enlarged. It was necessary to split the series modulo 12. for each of the resulting 12 series  $\mathbf{f}_i$ ,  $\mathbf{c}_i$  and  $\varphi_i$  were determined for the distinguishable peaks. 28 harmonic constituents were found unambiguously and for each of these the 12 separate estimates (or less when unanalysable) were used to find mean values and standard deviations of frequency, amplitude and phase. The results are listed in table III. The split series had a sampling interval of 3 hours, which caused some of the peaks to be "aliased" (ref. 11). The "corrected" frequencies of these peaks are listed in the table.

According to Doodson the astronomical tide H at any given time t may be written (neglecting nodal modulation)

$$H(t) = \sum_{i} H_{i} \cdot \cos(V_{i}(t) - g_{i}) = \sum_{i} H_{i} \cdot \cos(2 \cdot \pi \cdot f_{oi} \cdot t + V_{oi} - g_{i})$$
 (9)

where the V i's are linear combinations of six astronomical ele-

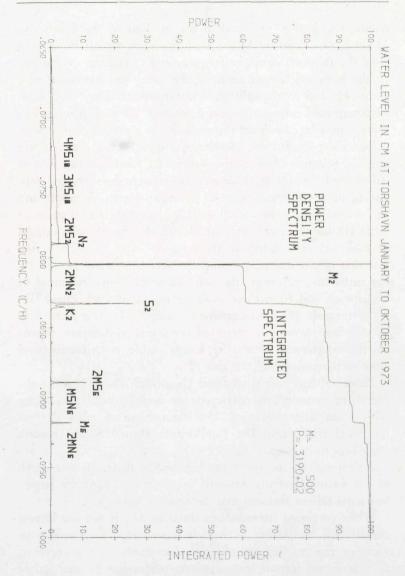


Fig. 3. MEM-spectrum and integrated spectrum of sea level fluctuations Tórshavn in the semidiurnal range. Vertical scale linear. Symbols of identified peaks listed. Some of the peaks are »aliased«.

ments  $\tau$ ,s,h,p,N,p' involved in the motions of the Earth and Moon in their orbits. As the elements vary almost linearly in time the (almost) constant frequencies  $f_{oi}$  emerge, giving rise to the harmonic constituents of the tide. The harmonic constants  $H_i$  and  $g_i$  (amplitude and Greenwich phase lag) must be determined empirically and separately for each place, while the  $f_{oi}$  may be calculated theoretically to a very high precision.

The most significant constituents have been given symbols and the identification of the harmonics found in the tide of Tórshavn is in almost all cases quite straight-forward by comparing observed frequencies to theoretical ones. Symbols and theoretical frequencies are listed in the last two columns of table III for those constituents identified. The phase constants of table III are relative to the time origin of the data series. For this origin one may, using the formulas cited in ref. 12, determine the values of the astronomic elements measured at Greenwich and from these evaluate the Greenwich Phase lags  $g_i$  of formula (9). Furthermore the amplitudes and phase lags of constituents may be corrected for nodal modulation, that is for the interference of smaller, unanalysable, constituents lying close in frequency (ref. 11 and 12).

Table IV lists amplitudes and Greenwich Phase lags for the identified constituents corrected for nodal modulation along with global characteristica. The last column of table IV lists the local time lags. The Equilibrium theory would require these lags to be zero.

A thorough discussion on the lags and on the relative strength of the various constituents will be given in a separate paper, but some salient features may be touched upon here.

When corrected for nodal modulation  $M_2$  is not the largest constituent, but rather  $O_1$  which is remarkable especially in view of the marked semidiurnal dominance in the currents. In a series of current measurements performed by the author in the summer of 1974 at a position only about 1 km distant from the tide gauge, a preliminary analysis of the current gave an amplitude for  $M_2$  nearly 12 times larger than that of  $O_1$ .

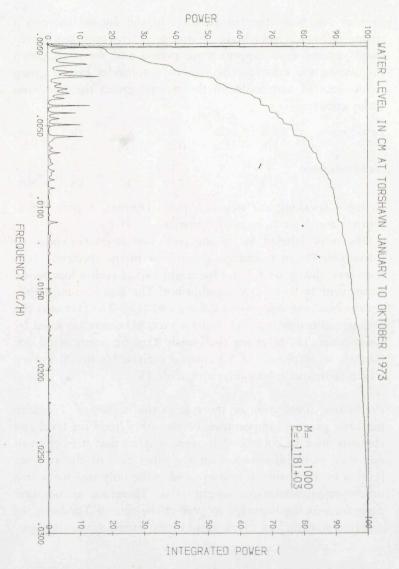


Fig. 4. MEM-spectrum and integrated spectrum of sea level fluctuations of Tórshavn in low frequency range. Vertical scale linear. The prominent peak is identified as Ssa.

Further one notes that in addition to the diurnal there is a strong influence from higher harmonics. Grouping the constituents according to approximate period and for each group calculating the ratio of the sum of amplitudes in the group to the sum of amplitudes in the diurnal group the following ratios appear:

| Period ratio:    | 24h  | 12h         | 8h   | 6h  | 4h  | 2.5h |
|------------------|------|-------------|------|-----|-----|------|
|                  | 12h  | 12 <b>h</b> | 12h  | 12h | 12h | 12h  |
| Amplitude ratio: | 1    | 1           | 1    | 1   | 1   | 1    |
|                  | 1.06 | 1           | 20.3 | 3.3 | 4.8 | 26.6 |

Roughly speaking the tide is 5 parts Diurnal, 5 parts Semidiurnal and 3 parts higher harmonics.

The peak labeled Ssa is the only low frequency peak of astronomic origin unambigously present in the spectrum. It is seen very clearly on fig. 4. One might expect such a longperiod constituent to be strictly Equilibrium. The Equilibrium amplitude is however only about 0.8 cm (ref. 13). The fact that Ssa is nonequilibrium is a well known fact, Maximov (as cited by Lisitzin ref. 13) gives for the North Atlantic north of 60 deg latitude an amplitude of 5.3 cm and a phase lag of 48 degrees, which compares favourably with table IV.

Prediction Prediction of the tide in the harbor of Tórshavn has little practical importance, on the other hand sea level and currents must be correlated in some way so that prediction of one may yield information on the other and of the two sea level is by far easier to measure and is the only one for which timeseries of reasonable length exist. Therefore a computer program was implemented to predict the tide in Tórshavn for a period starting any given time. In the program the six astro-

<sup>5.</sup> mynd. Samanburður millum spádda og mátaða flóð í Havn. Yvireftir er tíðin sett, frá 15. aug. 1974 kl. 0000 til 21. aug. 1974 kl. 23.45. Uppeftir er sett hæddin á flóðini. Munurin stavar serstakliga frá broytingum í lufttrýsti.

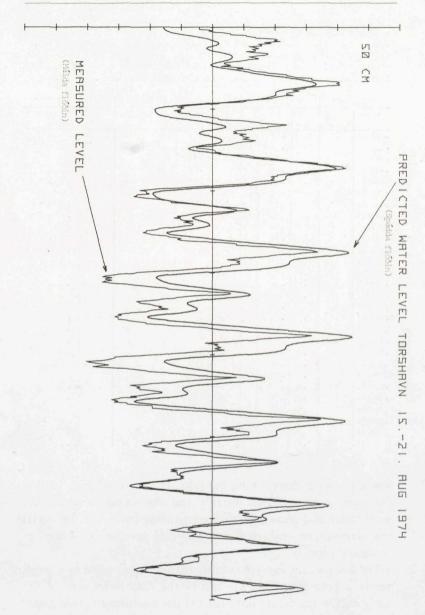


Fig. 5. Comparison between predicted and measured sea level Tórshavn. DC level of measured tide has been removed. Discrepancy between the two curves is mostly due to atmospheric pressure.

TABLE III Frequency, Amplitude and Phase values for harmonic constituents in the tide of Tórshavn. The Phase is relative to the time JAN. 1 st 1973 0000 GMT. Col. 5 and 8 list the number of determinations, which for each constituent have been used to estimate mean and standard deviation of frequency and amplitude, respectively phase. Aliased frequencies are "backaliased" in col. 9. The last two cols. list symbol and theoretical frequency for identified frequencies.

| FREQU   | JENCY               | AMPLITUDE |         | PHASE |       | FREQUENCY | SYMB. | FREQUENCY    |                   |             |
|---------|---------------------|-----------|---------|-------|-------|-----------|-------|--------------|-------------------|-------------|
| mean    | st dev              | mean      | st.dev. | no    | mean  | st. dev   | no    | corrected    |                   | theoretical |
| c/h     | 10 <sup>6</sup> c/h | cm        | cm      |       | rad   | rad       |       | c/h          |                   | c/h         |
| .000204 | .2                  | 6.52      | .04     | 6     | -1.11 | .01       | 5     |              | Ssa               | .000228     |
| .037213 | 18                  | 2.13      | .05     | 12    | -2.21 | .03       | 6     |              | Q <sub>1</sub>    | .037219     |
| .038756 | 5                   | 8.02      | .09     | 11    | .23   | .02       | 6     |              | 0,                | .038731     |
| .040230 | 41                  | -71-      | .05     | 9     | -2.46 | .24       | 8     |              | NO,               | .040243     |
| .041521 | 8                   | 1.37      | .05     | 6     | -2.20 | .14       | 4     |              | P <sub>1</sub>    | .041553     |
| .041791 | 12                  | 4.94      | .09     | 11    | -2.09 | .02       | 5     | Mind of the  | K <sub>1</sub>    | .041781     |
| .077654 | 22                  | 51        | .06     | 10    | 1.37  | .23       | 5     |              | 2MS <sub>2</sub>  | .077689     |
| .078981 | 10                  | 1.74      | .05     | 11    | 1.64  | .09       | 7     |              | N <sub>2</sub>    | .078999     |
| .080497 | 9                   | 10.25     | .07     | 9     | -1.97 | .02       | 8     | EL ALE       | M <sub>2</sub>    | .080511     |
| .081973 | 52                  | .68       | .06     | 9     | -2.73 | .04       | 4     | DEL TE       | 2MN <sub>2</sub>  | .082024     |
| .083325 | 17                  | 5.81      | .09     | 9     | 2.34  | .04       | 6     |              | S <sub>2</sub>    | .083333     |
| .083570 | 13                  | 1.61      | .05     | 14    | .07   | .16       | 7     |              | K <sub>2</sub> ·  | .083561     |
| .120771 | 19                  | .65       | .04     | 12    | -2.91 | .14       | 6     | and the same | M <sub>3</sub>    | .120767     |
| .122317 | 21                  | .42       | .05     | 9     | -1.80 | .21       | 14    |              | MK3               | .122292     |
| .155383 | 56                  | .46       | .07     | 6     |       |           |       |              |                   |             |
| .158117 | 53                  | .69       | .08     | 10    |       |           |       |              |                   |             |
| .159518 | 19                  | 1.31      | .12     | 10    | 1.32  | .08       | 5     |              | MNu               | .159511     |
| .161036 | 11                  | 3.02      | .06     | 12    | -2.42 | .01       | 14    |              | M <sub>11</sub>   | .161023     |
| .162473 | 50.                 | .58       | .06     | 8     |       |           |       |              | SNu               | .162333     |
| .163871 | 8                   | 2.24      | .11     | 9     | 2.00  | .06       | 5     |              | MS <sub>4</sub>   | .163845     |
| .130618 | 5                   | .40       | .05     | 5     |       |           | -     | .202715      |                   | The Market  |
| .094706 | 66                  | .51       | .06     | 7     |       |           |       | .238627      |                   |             |
| .093298 | 16                  | .85       | .03     | 11    | -1.63 | .01       | 6     | .240035      | 2MN <sub>6</sub>  | .240022     |
| .091776 | 15                  | 1.46      | .06     | 6     | .64   | .06       | , 5   | .241557      | Me                | .241534     |
| .090449 | 35                  | .66       | .08     | 9     | 2.24  | .23       | 6     | .242884      | MSN <sub>6</sub>  | .242844     |
| .088949 | 15                  | 1.67      | .13     | 7     | -1.61 | .04       | 4     | .244384      | 2MS <sub>6</sub>  | .244356     |
| .072087 | 33                  | .54       | .08     | 12    | .96   | .30       | 7     | .405420      | 4MS10             | .405379     |
| .074882 | 70                  | .48       | .06     | 7     | 65    | .66       | 5     | .408215      | 3MS <sub>10</sub> | .408201     |

nomic elements determining the tide are computed (ref. 12) for the starting time. Based on this and the values of table IV, amplitudes and phases of the constituents corrected for nodal modulation are evaluated and the tide computed — and if necessary plotted.

Fig 5 shows an example of prediction. The period is a week nearly a year later than the end of the data series used in the analysis. On the figure data for the period from the tide gauge have been plotted.

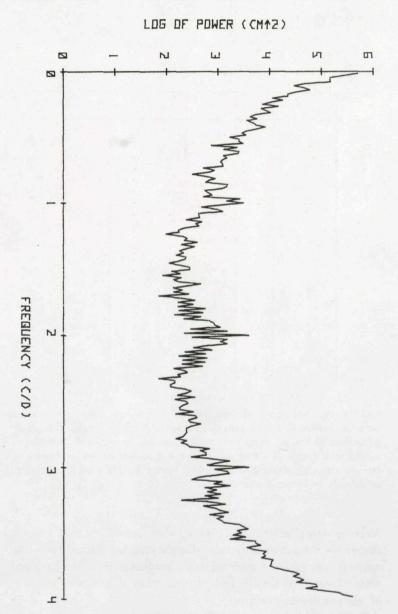


Fig. 6. Power spectrum (FFT) of reduced water level Jan-Oct 1973 Tórshavn. No filtering or correction made.

TABLE IV Harmonic constants for Tórshavn. The Greenwich Phase lag is the lag of the constituent relative to the phase of the relevant component of the astronomic driving force measured at Greenwich. The local time lag is the time that the constituent lags behind (in case of minus: leads) the driving force. Amplitude and phaselag have been corrected for nodular modulation. (ADD.IAG: Additional lag, GR.P.IAG: Greenwich Phase Lag,

| SYMBOL            | DOODSON NO.  | ADD: LAG | FREQUENCY | PERIOD | AMPLIT: | GR.P.LAG | L.T  | . LAG |
|-------------------|--------------|----------|-----------|--------|---------|----------|------|-------|
|                   | iojokolomono | deg.     | c/h       | h      | cm      | deg.     | h    | min   |
| Ssa               | 002000       | 180      | .000228   | 4386   | 6.31    | 85       | 1033 |       |
| Q <sub>1</sub>    | 1-2 0 1 0 0  | -90      | .037219   | 26.87  | 2.57    | 22       | 1    | 9     |
| 01                | 1-1 0 0 0 0  | -90      | .038731   | 25.82  | 9.98    | 57       | 3    | 37    |
| NO <sub>1</sub>   | 100100       | 90       | .040269   | 24.83  | .91     | -21      | -1   | 53    |
| P <sub>1</sub>    | 1 1-2 0 0 0  | -90      | .041553   | 24.07  | 1.36    | 116      | 7    | 17    |
| K <sub>1</sub>    | 110000       | 90       | .041781   | 23.93  | 4.84    | 130      | 8    | 12    |
| 2MS,              | 2-2 2 0 0 0  | 0        | .077689   | 12.87  | .47     | 87       | 2    | 38    |
| No 2              | 2-1 0 1 0 0  | 0        | .078999   | 12.66  | 1.79    | 175      | 5    | 40    |
| M <sub>2</sub>    | 200000       | -0       | .080511   | 12.42  | 9.89    | -164     | -6   | 8     |
| 2MN <sub>2</sub>  | 2 1 0-1 0 0  | 0        | .082024   | 12.19  | .65     | 53       | 1    | 20    |
| S <sub>2</sub>    | 2 2-2 0 0 0  | 0        | .083333   | 12.00  | 5.81    | -134     | -4   | 55    |
| K <sub>2</sub>    | 220000       | 0        | .083561   | 11.97  | 2.13    | -161     | -5   | 48    |
| M <sub>3</sub>    | 300000       | 180      | .120767   | 8.28   | .62     | 111      | 2    | 5     |
| MK <sub>3</sub>   | 3 1 0 0 0 0  | 90       | .122292   | 8.18   | .40     | -163     | -4   | 10    |
| MNu               | 4-10100      | 0        | .159511   | 6.27   | 1.30    | -85      | -1   | 57    |
| M <sub>4</sub>    | 400000       | - 0      | .161023   | 6.21   | 2.81    | -56      | -1   | 26    |
| MS <sub>u</sub>   | 4 2-2 0 0 0  | 0        | .163845   | 6.10   | 2.16    | -32      | -1   | 1     |
| 2MN <sub>6</sub>  | 6-1 0 1 0 0  | 0        | .240022   | 4.17   | .81     | 167      | 1    | 28    |
| M <sub>6</sub>    | 600000       | 0        | .241534   | 4.14   | 1.31    | -148     | -2   | 11    |
| MSN <sub>c</sub>  | 6 1-2 1 0 0  | 0        | .242844   | 4.12   | .66     | -138     | -2   | 2     |
| 2MS <sub>6</sub>  | 6 2-2 0 0 0  | 0        | .244356   | 4.09   | 1.55    | -102     | -1   | 37    |
| 4MS <sub>10</sub> | 10 2-2 0 0 0 | 0        | .405379   | 2.47   | . 35    | -85      | -1   | 3     |
|                   | 10 4-4 0 0 0 | 0        | .408201   | 2.45   | .43     | -75      | -0   | 58    |

TALVA IV. Aldingar í flóðini í Havn. Fyrstu trý røðini vísa almenn nøvn og eyðkenni, fjórða rað: Títtleikan (aldingar í tíman), fimta rað: Aldutíðina (í tímum), sætta rað: Styrkina (í cm), sjeynda rað: Seinkingina av aldingini í mun til sól og mána roknað í gradum og frá Greenwich og áttanda rað vísir seinkingina roknaða í tímum frá Havnini (— svarar til at aldingin er komin framum).

Meteorological influence Having determined — to a certain degree — the astronomic tide, this may be subtracted from the original time series. Referring to the resulting series as »Reduced water level« we may investigate to what degree this series is of meteorological origin.

In fig. 6 is shown the PDS of reduced water level and in

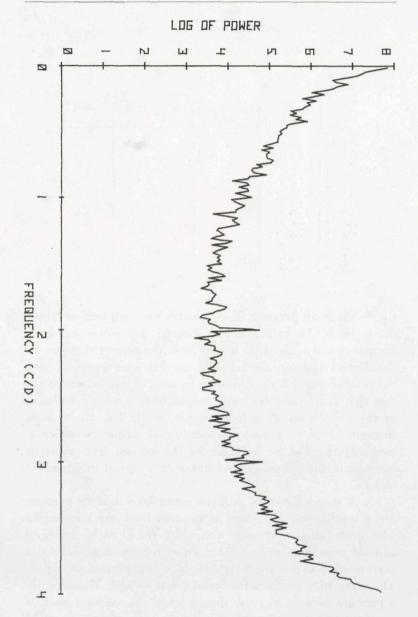


Fig. 7. Power spectrum (FFT) of air pressure Jan—Oct 1973 Tórshavn. No filtering or correction made.

 $\overline{\text{TABLE V}}$  The correlation between Sea level (reduced) and airpressure and the Barometer factor for separate months of 1973. Also listed the standard deviation of air pressure and the significance P, that is the probability of obtaining the observed barometer factor under the assumption that it has the theoretical value -1.01~cm/mb.

| PER | IOD | CORRELATION | BAROM.FACTOR cm/mb | ST.DEV.(air pr.) | P         |  |
|-----|-----|-------------|--------------------|------------------|-----------|--|
| TAN | 70  | 0.000       |                    | 41, 20           | 0.01      |  |
| JAN | 73  | -0.948      | -0.788             | 14.79            |           |  |
| FEB | 11  | -0.886      | -0.732             | 14.87            | "         |  |
| MAR | 11  | -0.903      | -0.880             | 13.32            | 11        |  |
| APR | 11  | -0.924      | -0.720             | 12.13            | 11        |  |
| MAY | 11  | -0.688      | -0.489             | 9.36             | "         |  |
| JUN | 11  | -0.851      | -0.920             | 8.67             | "         |  |
| JUL | 11  | -0.868      | -1.024             | 5.91             | Not Sign. |  |
| AUG | 11  | -0.931      | -0,899             | 9.32             | 0.01      |  |
| SEP | 11  | -0.941      | -0.898             | 10.32            | "         |  |
| OCT | 11  | -0.858      | -0.931             | 8.16             | 0.05      |  |
| NoA | 11  | -0.879      | -0,803             | 14.21            | 0.01      |  |
| DEC | 11  | -0.798      | -0.689             | 11.52            | 11        |  |

fig. 7 that of air pressure. These spectra have not been obtained using the MEM, rather a Fast Fourier Transform has been computed and squared. It is seen from the spectra that not all the diurnal and semidiurnal influence has been removed from the reduced water level (this may to some degree be due to the fact that corrections for nodal modulation were not included in the calculation of reduced water level). The air pressure spectrum displays a peak at semidiurnal frequency which is probably the  $S_2$ , the influence of this on sea level must be intrinsic in the amplitude and phase of  $S_2$  given in tables III and IV.

Fig. 8 shows a plot of reduced water level and air pressure for a specific month chosen at random from the data series. The interrelation is obvious. Taking all the available sea level and air pressure data in 1973 a linear regression analysis was performed between the two variables and plotted in fig. 9. The correlation coefficient obtained was —0.82. Theoretically a pressure increase of 1 mb should lower the sea level by 1.01 cm. In contrast to this theoretical barometer factor of —1.01

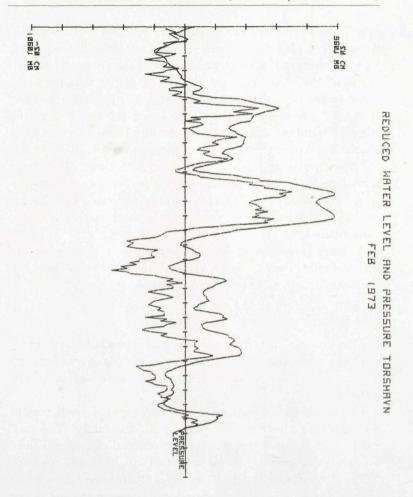


Fig. 8. Comparison of reduced water level and air pressure Tórshavn Feb. 1973. Vertical scales are adjusted such that 1 mb corresponds to — 1 cm. 8. mynd. Munurin millum spádda og mátaða flóð februar 1973 (niðara kurvan) samanborin við barometurtrýstið (teknað at vaksa niðureftir) somu tíð (ovara kurvan). Á myndini svarar 1 millibar (mb) til 1 cm.

cm/mb the regression analysis yielded a value of —0.73 cm/mb. A possible explanation for the discrepancy emerges from table V, which lists results of regression analysis performed for each month of 1973 separately. In all but one of the cases the barometer factor is significantly different from the theoretical one at the 5 % level and this one case is for the month with the minimal standard deviation of pressure. There is a tendency for the discrepancy to grow with the pressure variation as fig. 10 demonstrates and this is interpreted as follows. With large variation in pressure — hence large st. dev. — a comparatively larger amount of the variation may be too rapid for the sea level to be able to respond, resulting in a lower barometer factor than theoretical.

A closer investigation into this hypothesis is in progress and will together with a more detailed description of the low frequency spectrum be presented elsewhere.

#### Conclusions

Although no actual comparison has been made with other methods, it has been demonstrated, that the Maximum Entropy Method is well suited to perform quantitative anlysis of the astronomic tides.

The tabulation of harmonic constants permits prediction of the astronomic tide of Tórshavn quite accurately. The prominence of higher harmonics in the tide indicate strong nonlinearities presumably due to frictional effects in the currents.

The nonharmonic part of sea level fluctuation seems to be mostly due to air pressure variations, however the sea is not found to be able to respond to rapid pressure fluctuations.

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Most of this work has been done while the author was recieving a grant from Nordisk Kollegium for Oceanografi and on the Institute of Physical Oceanography at the University of Copenhagen headed by Prof. Jerlov. The inspiration of Dr

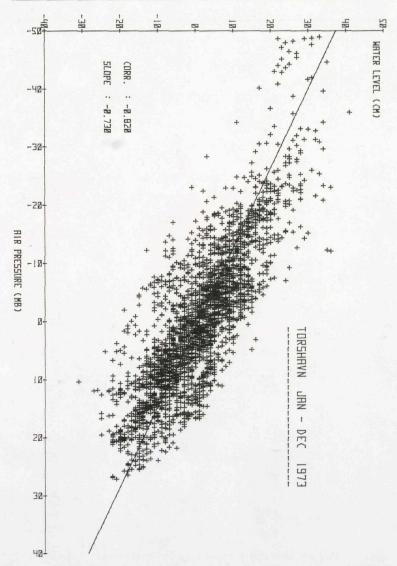


Fig. 9. Regression between reduced water level and air pressure Tórshavn 1973. Numbers on axes are deviations from mean. Listed is the correlation coefficient and the slope of the regression line.

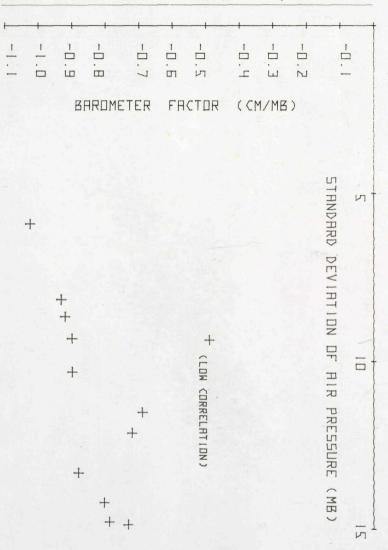


Fig. 10. Plot of calculated barometer factor versus standard deviation of air pressure for separate months of 1973 in Tórshavn.

Jerlov and Dr Kullenberg and discussions with other members of the Institute are most heartily appreciated.

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#### SAMANDRÁTTUR

Í nøkur ár hevur ein flóð- og fjørumátari hjá danska Meteorologiska stovninum staðið í Havnini, sum hvørt korter hevur mátað hæddina á vatnskorpuni, t. e. flóðina. Í hesi grein verður greitt frá nøkrum fyribils útrokningum við tølum úr tíðarskeiðnum jan.—okt. 1973.

Broytingin av flóðini h í tíð er lýst í líkningini (1). Fyrsta lið á høgru síðu á (1) er tann parturin av flóðini sum stavar frá sól og mána. Hann kann uppfatast sum samansettur av fleiri reglubundnum aldingum við hvør sínum títtleika, hvør síni styrki og hvør síni seinking (f $_{\rm i}$ , c $_{\rm i}$  og  $\varphi$  $_{\rm i}$ ). Harumframt er ein partur h $_{\rm p}$  sum stavar frá broytingum í lufttrýsti og ein rest h $_{\rm R}$  sum ilt er at greina nærri.

Við einum roknihátti, ið nevndur verður Maximum Entropy Method ber til burturúr mátaðu flóðini at rokna seg fram til hvørjar aldingar eru til staðar og hvønn títtleika og hvørja styrki og seinking hvør einkult teirra hevur. Úrslitini eru sett í talvu III og IV.

Tølini kunnu býtast í fýra høvuðsbólkar eftir hvussu langa tíð aldingin hevur um at endurtaka seg. Fyrsti bólkur umfatar heilt spakuligar aldingar, og har var bert ein funnin við hálvum ári í aldutíð. Annar og triði bólkur hava aldutíðir, ið eru ávikavist heilan og hálvan dag, og í fjórða bólki eru knappari broytingar. Allar gera tær sítt til at ávirka hæddina á vatnskorpuni, men skilliga sæst at serstakliga hálvdagligu og dagligu aldingarnar eru av týdningi.

Við at nýta tølini í talvu IV ber til at spáa um flóðina til ávísa tíð. Mynd 5 vísir eitt dømi, her er spádda flóðin teknað saman við mátingum frá sama tíðarskeiði.

Munurin millum spáddu og mátaðu flóðina vísir seg fyri stóran part at stava frá broytingum í lufttrýsti. Á mynd 8 er hesin munur teknaður fyri februar mánað 1973 saman við lufttrýstinum somu tíð (roknað at vaksa niðureftir), og tað sæst at ein vøkstur í trýsti førir við sær at flóðin minkar. Tó nær sjógvurin ikki at fylgja við um trýstið broytist ov knappliga.