In Situ Determinations and Statistical Analysis of Magnetic Susceptibilities of Basalts of the Faroe Islands

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Abstract.

The concept of magnetic susceptibility is shortly reviewed in the light of geophysics. A susceptibility meter is described and the basis for design of the instrument is examined.

200 determinations of magnetic volume susceptibility of the Faroese basalts are presented. Generally, it can be said that basalts of the lower series seem to have a higher susceptibility than basalts of the middle series. A statistical analysis shows that with a level of significance of 5 % the 200 determined values follow a logarithmic normal distribution. It is thus proposed that normally susceptibility data may be analysed by the logarithmic normal distribution.

Introduction.

In the summer of 1972 the Laboratory of Geophysics at the Department of Geology, Århus University, took part in an international seismic project in the North Atlantic region. In connection with the stay on the Faroe Islands the authors had the opportunity to make a series of in situ determinations of magnetic volume susceptibility of the Faroese basalts. Observations were made in selected localities on Mykines, Viðoy, Vágar and Suðuroy, and can be regarded as preliminary.

For several years the Laboratory of Geophysics has been making geophysical measurements on and around the Faroes. A summary has been made by *Saxov* (1970). The first magnetic measurements were made by Abrahamsen (*Saxov & Abrahamsen,* 1966), who has also made palaeomagnetic studies on the Faroes (*Abrahamsen,* 1967). Later on the measurements have been followed up by marine magnetic measurements in the waters around the islands (*Schrøder,* 1971). The first in situ determinations of magnetic susceptibility were made by Abrahamsen on 14 lava flows from the lower series in the main-profile section Hvannafelli, and the results are shortly described in *Saxov* (1970).

Magnetic Susceptibility.

In the following the concept of susceptibility will be shortly described in the light of geophysics. Consider a material placed in a magnetic field $\bar{H}$. The material is magnetized, and let the induced magnetic moment per unit of volume de disignated by $\bar{J}$. If the material is isotropic[1]) the magnetic volume susceptibility is defined thus:

$$\kappa = \bar{J}/\bar{H}$$

In geophysics the electromagnetic centimetre-gramme-second (e.m.c.g.s.) system of units is traditionally used. In this system $\bar{J}$ is measured in abampere per cm per cm^3 and $\bar{H}$ is measured in oersted. The dimension of the volume susceptibility is thus cm^{-3} (concerning units see footnote[2])).

For paramagnetic materials κ can be taken as a constant sized between 10^{-7} and 10^{-4} Important paramagnetic minerals are olivine, pyroxene, amphibole, granate and biotite.

For diamagnetic materials κ can also be taken as a constant

[1]) Many rocks are highly anisotropic. In this case the susceptibility cannot be said to be the same size for all directions in the rock.

[2]) 1 abampere (absolute ampere) = 10 »ordinary« ampere

$$1 \text{ oersted} = \frac{1}{4\pi} \times \text{abampere per cm.}$$

and of the same order of magnitude, numerically, only negative; i. e. the induced magnetic moment is directed oppositely the external field. Quartz, feldspar, rock-salt and gypsum are diagmagnetic.

Certain iron-titanium minerals whose composition falls within the ternary system FeO-Fe_2O_3-TiO_2 have quite different properties. They have a positive and considerably larger susceptibility, which is not a constant but depends on the temperature and the external field $\bar{H}$. For these minerals $\bar{J}$ will, as a function of $\bar{H}$ (at a given temperature), take the form of a so-called hysteresis loop, a non-linear curve. When the minerals are heated to above the Curie-temperature, which for magnetite is 578° C, they get paramagnetic. Minerals with the properties described here are collectively called ferromagnetic.

Another important property of ferromagnetic minerals is their ability to possess a permanent magnetic moment, independent of the external field. The study of this is the subject of the science of palaeomagnetism.

The most important ferromagnetic minerals are magnetite, titanomagnetite and ilmenite, and it is primarily the content of these minerals which is responsible for the susceptibility of most rocks, in spite of the fact that they only amount to a few percent of common rocks. You can say that the dia- and paramagnetic properties of the other minerals are »hidden« behind the ferromagnetic properties. As for the Faroese basalts it is the content of magnetite and ilmenite which is responsible for the susceptibility.

Measuring Procedure and Instrument Description.

The susceptibility meter which was used is Finnish and constructed by the Geological Survey of Finland. The basic circuit of the instrument is a resonance bridge. Three branches of the bridge are pure ohmic resistances, one of which is variable. The fourth branch, connected parallelly to the variable resistance, consists of a fixed condenser, connected in series

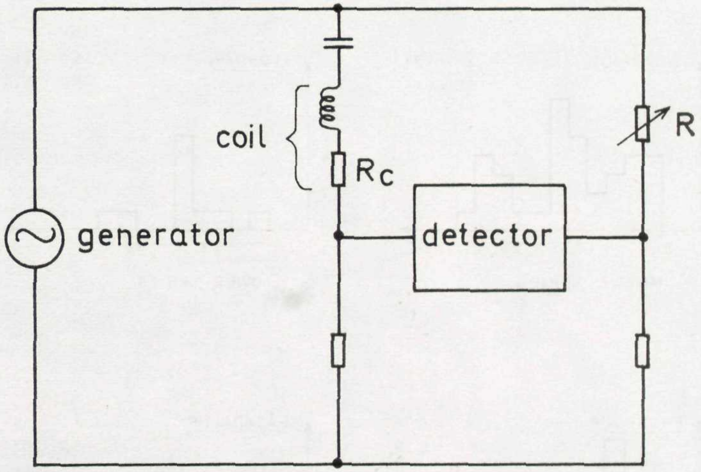

Fig. 1. *Bridge Circuit.*

with a coil with resistance R_c — this coil is the sensing probe. The voltage is obtained from an A. C. generator with variable oscillation frequency. The sensitivity of the instrument is 15.4 $\times 10^{-6}$ emu/cm³.

The principle of a susceptibility measurement is as follows: The sensing probe is held in the air and the frequency of the generator is set on a value equivalent to the resonance frequency of the L-C branch. The impedance of the L-C branch is now of pure ohmic nature, and balance is achieved by adjusting the variable resistance to the resistance R_c of the L-C branch. Generally, this adjustment can be maintained, except when very conducting material is placed in the proximity of the sensing probe. Now the sensing probe is approached to and held at a given height above the material, the susceptibility of which is to be determined. The self inductance of the coil is changed and in consequence the resonance frequency of the L-C branch changes. Balance is attained by readjusting the frequency of the generator to the new resonance frequency.

It can be shown that for small κ ($\kappa << 1$) a relative

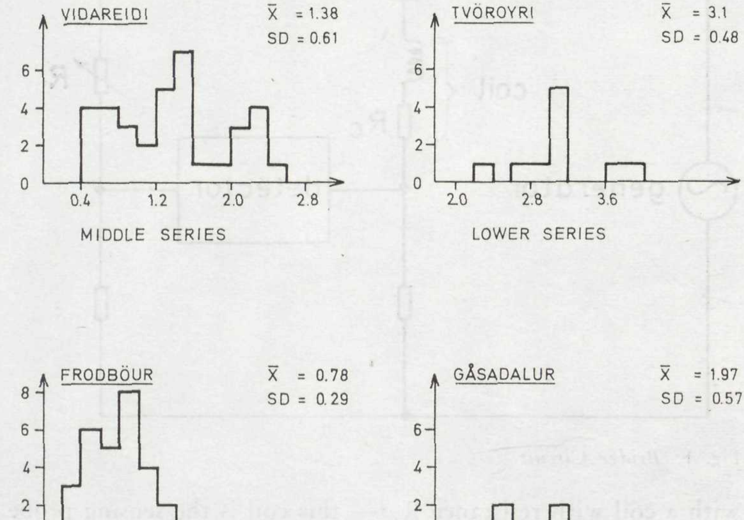

Fig. 3. *Histograms of Volume-Susceptibilities for all localities.*
Abscissa: Volume-Susceptibility x 10^{-3} *emu/cm*^{3} *— Class width 0.2 x* 10^{-3}
Ordinate: Absolute frequency.
$\overline{X}$ *= arithmetric mean.*
SD = standard deviation.

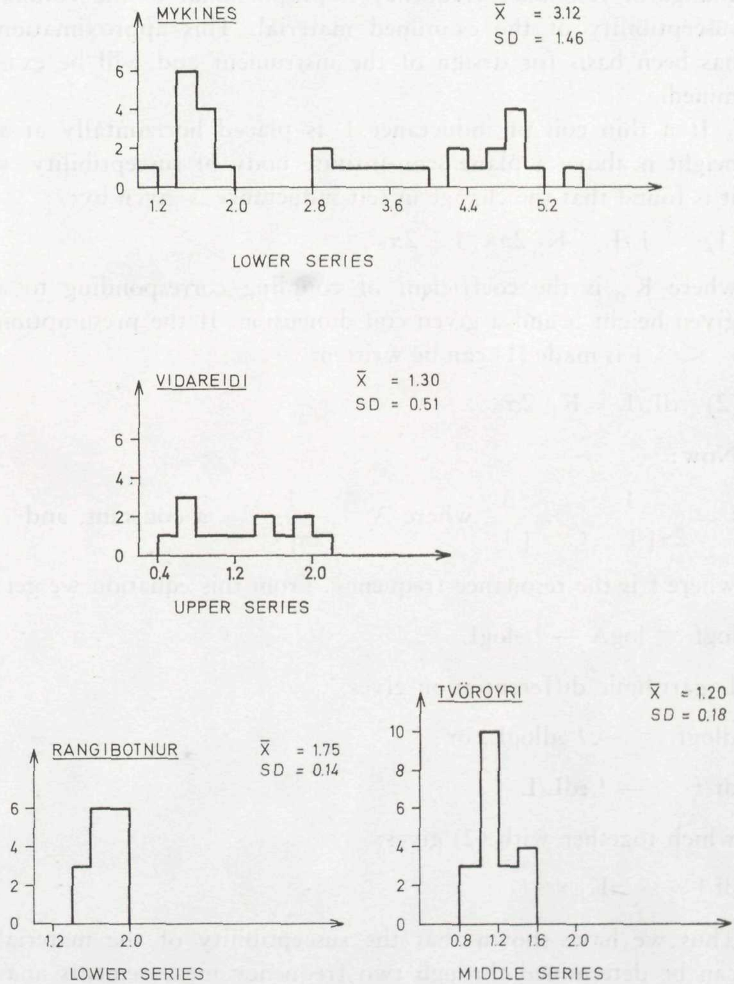

Fig. 3. *Continued*

change of resonance frequency is proportional to the volume susceptibility of the examined material. This approximation has been basis for design of the instrument and will be examined.

If a thin coil of inductance L is placed horizontally at a height h above a plane semi-infinite body of susceptibility κ it is found that the change in self inductance is given by:

(1) $\triangle L/L = K_h 2\pi\kappa / 1 + 2\pi\kappa$,

where K_h is the coefficient of coupling corresponding to a given height h and a given coil dimension. If the presumption $\kappa << 1$ is made (1) can be written:

(2) $dL/L = K_h\ 2\pi\kappa$

Now:

$$f = \frac{1}{2\pi\sqrt{L \times C}} = \frac{A}{\sqrt{L}}, \text{ where } A = \frac{1}{2\pi\sqrt{C}} = \text{ a constant, and}$$

where f is the resonance frequency. From this equation we get:

$\log f = \log A - {}^{1}/_{2}\log L$

Logarithmic differentiation gives:

$d\log f = - {}^{1}/_{2} d\log L$, or

$df/f = - {}^{1}/_{2} dL/L$

which together with (2) gives:

$df/f = -\pi K_h \kappa$

Thus we have shown that the susceptibility of the material can be determined through two frequency measurements analogous to the description above.

Experimentally K_h for the used coil has been determined to be 0.649 for h = 8 mm.

Presentation and Statistical Analysis of the Data.

The localities where the investigations were carried out is

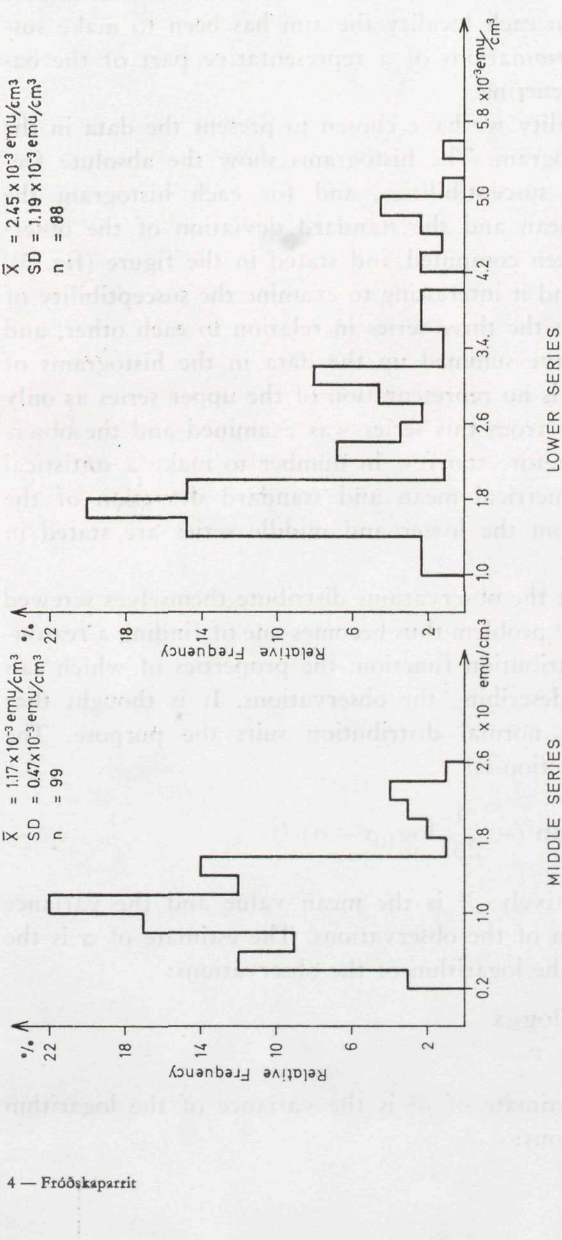

Fig. 4. *Histograms of Susceptibilities for lower and middle Series.*
Class width: 0.2 x 10^{-3} emu/cm³.
$\overline{X}$ = *arithmetic mean.*
SD = standard deviation.
n = number of observations.

marked on the geological map of the Faroes (fig. 2). It is seen that most of the localities are situated in the lower and middle basalt series. On each locality the aim has been to make susceptibility determinations of a representative part of the basalt varieties occurring.

For each locality we have chosen to present the data in the form of a histogram. The histograms show the absolute frequency of the susceptibilities, and for each histogram the arithmetrical mean and the standard deviation of the observations have been computed and stated in the figure (fig. 3).

We have found it interesting to examine the susceptibility of the basalts from the three series in relation to each other, and therefore we have summed up the data in the histograms of figure 4. There is no representation of the upper series as only a single locality from this series was examined and the observations are therefore too few in number to make a statistical analysis. Arithmetrical mean and standard deviation of the observations from the lower and middle series are stated in the figure.

It is seen that the observations distribute themselves screwed to the right. The problem thus becomes one of finding a reasonably simple distribution function, the properties of which can be used when describing the observations. It is thought that the logarithmic normal distribution suits the purpose. The distribution function is:

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp\left(-\frac{1}{2\beta^2}\log_{10}x - \alpha)^2\right)$$

where α respectively β^2 is the mean value and the variance of the logarithm of the observations. The estimate of α is the mean value of the logarithm of the observations:

$$\hat{\alpha} = \bar{u}_{\log_{10}} = \frac{\Sigma \log_{10} x}{n}$$

Similarly the estimate of β^2 is the variance of the logarithm of the observations:

$$\beta^2 = Su^2 = \frac{\Sigma(u-\bar{u})^2}{n-1}, \text{ where}$$

$$u = \log_{10}x$$

In figure 5 the susceptibility values of the lower and middle series are represented in the form of histograms with logarithmic abscissa. Similarly figure 6 is a histogram of all observations.

Now it is of interest whether the observed distributions with a certain significance level can be said to follow a logarithmic normal distribution, or whether there is a significant difference. For this purpose a chi-square test is used. By means of the chi-square test the hypothesis is tested: the observed distribution differs only »at random« from the logarithmic normal distribution. The result is summarized in table II, and it is seen that for all observations together and for the observations from the middle series the hypothesis is acceptable with a significance level of 5 %. For the observations from the lower series the hypothesis is rejected with the same level of significance.

To give a visual impression of the fit of the observed distributions in relation to the logarithmic normal distribution the graphs of the theoretical distributions have been put into the histograms. For the theoretical parameters the calculated estimates have been used. It is noticed that the observed distributions generally are too »peaked«.

The calculation scheme for the chi-square test for all observations together is stated in table I. In table II the results of the statistical analysis are summed up, and table III shows the result of calculations made for content of magnetite and ilmenite of the basalts on the basis of chemical analysis (*Rasmussen & Noe-Nygaard*, 1969).

Discussion and Conclusions.

The present statistical analysis gives reason to presume as a preliminary hypothesis that the magnetic volume susceptibility

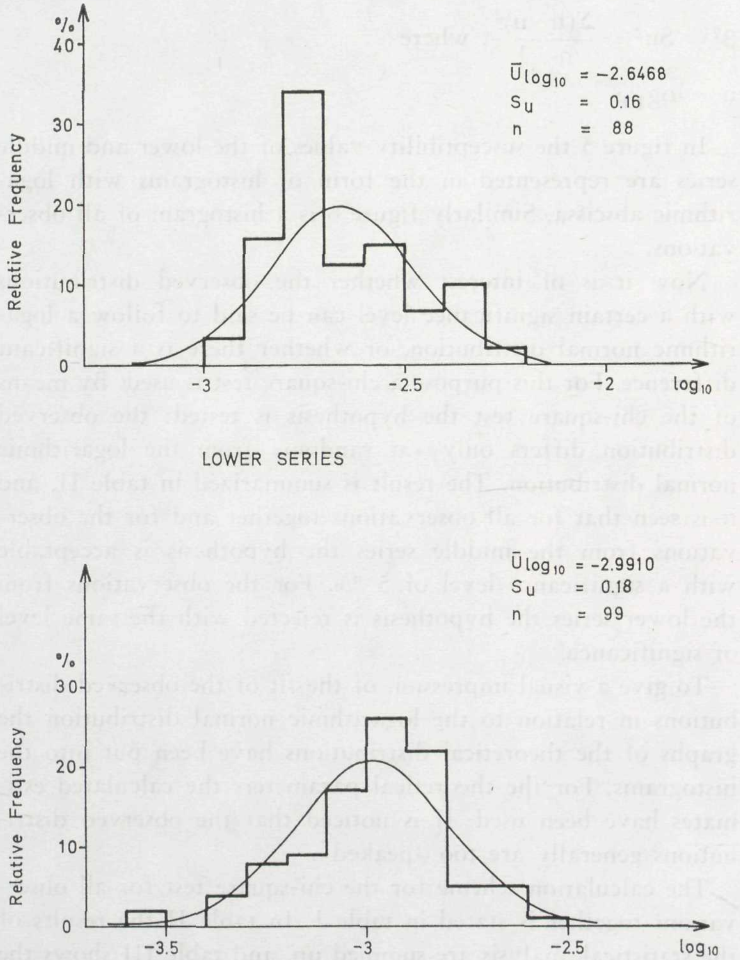

Fig. 5. *Histograms of Susceptibilities for middle and lower Series. The Abscissa is logarithmic (to the Base 10).*

$\bar{U}_{log_{10}}$ = *logarithmic mean.*

S_u = *logarithmic standard deviation.*

n = *number of observations.*

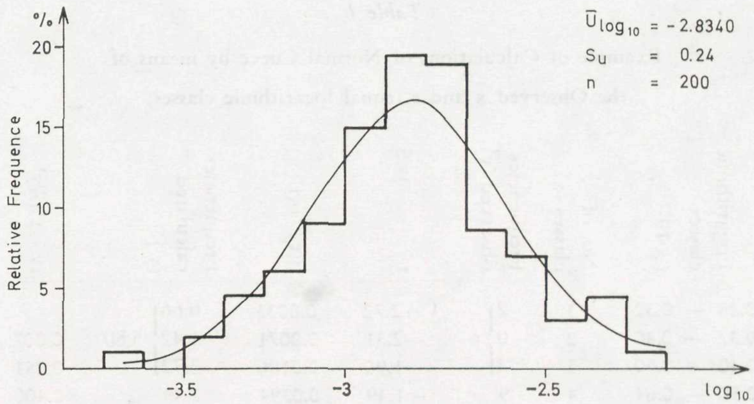

Fig. 6. *Histogram of Susceptibilities — Basalts of the Faroe Island. The Abscissa being logarithmic (to the Base 10).*

$\overline{U}_{log10}$ = *logarithmic mean.*

S_u = *logarithmic standard deviation.*

n = *number of observations.*

for basalts of the Faroe Islands can be seen collectively as logarithmically normal distributed. Naturally, the determinations of the upper series are very few in number, but there is no reason to presume that the hypothesis would not also hold good if additional observations of this series were included. The basalts of this series do not differ much from the basalts of the other series (cf. statistical analysis, *Saxov* (1970), mentioned below), and the content of magnetite is by and large as it is in the basalts of the middle series, the content of ilmenite being a little smaller (cf. table III).

The fact that the observations from the lower series cannot be described by a logarithmic normal distribution is no doubt due to the fact that the data do not represent a random sample. In the chosen localities we could only gather observations in vertically limited areas, so that the data represent only a few well defined levels in the stratigraphical column. Furthermore the samples were often taken from the top part or even from the surface of the lava flows. This is unfortunate if a gravi-

Table I

Example of Calculations of Normal Curve by means of the Observed $\bar{x}$ and s (equal logarithmic classes)

Logarithmic classes ($\times 10^{-3}$)	No. of classes (x)	Frequencies observed (y)	$t = (x_i - \bar{x})/S$	$\phi(t_i) - \phi(t_{i-1})$	Frequencies calculated (y')	$(y - y')^2/y'$
			$-\infty$			
0.25 — 0.32	1	2	−2.72	0.0033	0.66	
0.32 — 0.40	2	0 } 6	−2.31	0.0071	1.42 } 5.80	0.007
0.40 — 0.50	3	4	−1.90	0.0186	3.72	0.081
0.50 — 0.64	4	9	−1.49	0.0394	7.88	0.400
0.64 — 0.80	5	12	−1.08	0.0720	14.40	0.815
0.80 — 1.01	6	18	−0.67	0.1113	22.26	0.002
1.01 — 1.28	7	30	−0.26	0.1460	29.20	1.010
1.28 — 1.60	8	39	0.16	0.1660	33.20	2.018
1.60 — 2.03	9	38	0.58	0.1554	31.08	2.296
2.03 — 2.56	10	17	0.98	0.1225	24.50	0.309
2.56 — 3.21	11	14	1.39	0.0812	16.24	1.159
3.21 — 4.08	12	6	1.80	0.0464	9.28	
4.08 — 5.12	13	9 } 11	2.21	0.0223	4.46 } 7.18	2.032
5.12 — 6.40	14	2	$+\infty$	0.0136	2.72	
						$\chi^2 = 10.129$

k = 11 (number of classes with $y > 5$)
c = 2 (number of estimated parameters)
f = k — 1 — c = 11 — 1 — 2 = 8 (number of degrees of freedom)
$P(\chi^2 \leq 15.51) = 95\%$ for f = 8

tative differentiation has taken place in the comparatively thick lava flows of this series.

Our determinations seem to show that generally the values of the lower series are higher than the values for the middle series. It is difficult to make a statistical analysis to show this as the data of the lower series do not seem to follow a simple distribution. Thus it is hard to describe the two sets of data on a common basis.

An arithmetic mean of 2.2×10^{-3} emu/cm³ (*Saxov*, 1970)

Table II

Statistical Analysis of Susceptibility Data from the Faroese Basalts

	N	$\bar{X}$	SD	$\bar{U}_{log}$	$\bar{U}_{geom}$	S_u	χ^2	f		
Upper series ...	13	1.30	—	—	—	—	—	—	—	—
Middle series ...	99	1.17	0.47	—2.9910	1.02	0.16	9.01	5	$P(\chi^2 \leqq 11.70) = 95\%$	acceptable
Lower series ...	88	2.45	1.19	—2.6468	2.26	0.18	19.07	3	$P(\chi^2 \leqq 7.81) = 95\%$	not acceptable
All observations	200	1.61	1.11	—2.8340	1.47	0.24	10.13	8	$P(\chi^2 \leqq 15.51) = 95\%$	acceptable

N = number of observations
$\bar{X}$ = arithmetic mean ($\times 10^3$ emu/cm^3)
SD = standard deviation ($\times 10^3$ emu/cm^3)
$\bar{U}_{log}$ = logarithmic mean
$\bar{U}_{geom}$ = geometric mean ($\times 10^3$ emu/cm^3)
S_u = logarithmic standard deviation
χ^2 = chi-square value
f = degrees of freedom

Table III

Content of Magnetite and Ilmenite of the Faroese Basalts on the basis of Chemical Analyses (Rasmussen & Noe-Nygaard, 1969)

	Magnetite			Ilmenite		
	$\bar{X}$	SD	N	$\bar{X}$	SD	N
	%			%		
Lower series	9.3	3.7	35	5.0	1.4	35
Middle series	6.8	1.7	35	4.2	1.6	35
Upper series	5.7	1.8	23	2.6	0.9	23

has been calculated for Abrahamsen's determinations of susceptibility of the lava flows of Hvannafelli (lower series). This is in agreement with the results of our survey.

Saxov has made an extensive statical analysis on the basis of chemical analysis of the basalts (*Rasmussen & Noe-Nygaard*, 1969). The conclusion is that there is no significant difference in the chemical composition of the basalts of the three series (*Saxov*, 1970). However, a calculation of the average content of the minerals magnetite and ilmenite seems to show a certain discrepancy between the basalts of each series. For the lower series the content of magnetite seems to be higher than for the other series, which confirms the fact that generally the susceptibility values for the lower series are higher than for the middle series. The content of ilmenite seems to be higher for the lower series too.

Irving et al. (1966) have examined the susceptibility af sandstone from the Scottish Caledonians, and they have reached the conclusion that the logarithmic normal distribution is a suitable basis for description of susceptibility data. On a theoretical basis *Janák & Uman* (1967) concluded that susceptibility data for rocks with low values would distribute themselves asymmetrically.

In *Tarling* (1971) it is stated that susceptibility in any rock is distributed in a logarithmically normal form, and it is concluded that the main influence on susceptibility is the distribution of grain sizes as these are log-normally distributed.

The results we have obtained for the Faroese basalts are in good agreement with the above cited conclusions.

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ÚRTAK

Grein er í stuttum gjørd á hugtakinum sigulmagnsviðkvæmi í ljósi frá jarðalisfrøði. Eitt viðkvæmismát er lýst, og støðið undir tilevning av tólinum er kannað.

200 kanningarúrslit av sigulmagnaðum royvisviðkvæmi hjá føroyskum basalti verða løgd fram. Alment kann verða sagt, at niðastu basaltfláirnar tykjast viðkvæmari enn miðfláirnar. Hagfrøðilig sundurgreining sýnir, at við eini týdningshædd upp á 5 % fylgja tey 200 funnu úrslitini einari logaritmiskt rættvorðnari sundurbýting. Tí verður skotið upp, at úrslit frá viðkvæmismátingum verða vanliga greind eftir tí logaritmiskt rættvorðnu sundurbýtingini.